

# **A methodology using option pricing to determine a suitable discount rate in environmental management**

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**ABSTRACT.** With the present analysis the authors propose an approach for determining an adequate discount rate in environmental management problems, more specifically radioactive waste management. It is shown that the classical Black-Scholes pricing formula can be used for determining the adequate present funding to be set aside for the future. The average funding is equal to the net present value (NPV) of the future costs, including technical-scenario uncertainties. For taking into account the financial uncertainties, the NPV is identified with the strike price of a European put option, and the asset value in the managed fund is identified with the current price. The discount rate to be determined is identified with the risk-free interest rate. The paper shows that the adequate present funding can be determined for given multi-generational risk levels and an asset allocation by fixing the discount rate and adding a premium to the NPV of future costs.

## **1. SCOPE OF THE ANALYSIS**

The choice of a discount rate for comparing environmental costs distributed over time is a difficult

task, because of the multi-generational impacts of long-term externalities. The literature on this topic is vast, but a universal approach reconciling in any situation the economics with the multi-generational aspects does not seem to exist. With the present analysis the authors propose an approach for determining an adequate discount rate in the specific case where funding in the present supports the future generations in removing the long-term externalities. This methodology has been used by ONDRAF/NIRAS the Belgian agency for radioactive waste management for determining the right level of funding.

## **2. BASIC ASSUMPTIONS OF THE MODEL**

In this note we consider a general nuclear-waste-management project to be properly funded at the inception time, i.e., just before the project starts. It is assumed that the overall budget of the project, including uncertainty margins, is completely known. The problem is to evaluate the discount rate (d.r.) for calculating the Net Present Value of the total cost distributed over an extended time horizon, which is

also assumed to be known. The funding is to be set aside in a fund, managed according to a specific strategic asset allocation (AA).

To make things simple in this methodological paper, we will compare two AA's in the following, using constant currency units at the date  $t=0$ , and thus real rates of return:

- A low-risk portfolio with 100% bonds. The couple (return=2,30%; standard deviation=3,90%) is estimated from historical data series;
- A medium-risk portfolio with 50% equities/50% bonds. The couple (return=5%; standard deviation=10%) is estimated from historical data series.

(Note that a an environment agency being often the property of the State would generally not be willing to implement a high-risk AA with 75% to 100% equities).

Of course many other assumptions can be used.

Further the assumption is made for the asset evolution of brownian motion, so that the resulting distribution of the asset price  $S$  is lognormally distributed at  $t=T$ .  $T$  is the portfolio liquidation date.

We consider several  $T$ -values, and we examine in each case the funding ratio. We will be using the values  $T=5, 15, 25, 50, 100$  years, which are typical for radioactive-waste management.

More formally,  $\ln S(T)$  is normally distributed (see Hull 2000; Beninga 2000):

$$\Phi \left[ \ln S_0 + (\mu - \sigma^2 / 2)T, \sigma\sqrt{T} \right] \quad (1)$$

, where  $\ln S_0$  represents the value of the asset in  $t=0$ ;  $\Phi(m, s)$  represents the cumulated normal distribution of mean value  $m$ , and standard deviation  $s$ .

Under those conditions, we can use the theory of financial options, i.e., here simply the Black-Scholes formula, to calculate the probability that the asset  $S(T)$  will be superior (inferior) to the liability covered by the funding.

### 3. EQUILIBRIUM IN THE FUNDING AT THE EXERCISE TIME T

In the following we assimilate the spot price  $S_0$  in  $t=0$  to the assets placed in the fund at the inception date in  $t=0$ . In the same way we assimilate the strike price  $X$  in  $t=T$  (exercise time) to the future value of the total liability of the fund at  $T$ . Remember that in this paper we consider only financial uncertainties, so we assume

that all technical uncertainties have been taken into account when calculating the  $X$ -value.

Calling a the discount rate (d.r.), the funding at start in  $t=0$  to have the equality asset = liability will be:

$$S_0 = Xe^{-aT} \quad (2a)$$

('initial-funding' equation)

The available assets in  $t=T$  are represented by the lognormal distribution defined in (1). Adequate funding situations in  $T$  will impose:

$$S(T) \geq X \quad (2b)$$

('adequate-funding in  $T$ ' equation)

Because  $S$  is a stochastic variable, lognormally distributed according to (1), this equation will be verified for a given confidence level. Taking for example a confidence level 95% means that, with a probability of 95%,  $S_0$  must be larger than  $X$ , i.e.:

$$\ln S_0 + (\mu - \sigma^2 / 2)T - k\sigma\sqrt{T} = \ln X \quad (3)$$

$$k = \Phi^{-1}(95\%) = 1.645$$

, where  $\Phi^{-1}$  is the inverse cumulative normal distribution of mean=0 and standard deviation=1. This implies under equation (2a):

$$\ln(Xe^{-aT}) + (\mu - \frac{\sigma^2}{2})T - k\sigma\sqrt{T} = \ln X \quad (4)$$

$$a = (\mu - \frac{\sigma^2}{2}) - k\sigma/\sqrt{T}$$

The use of the d.r. (4) will thus guarantee the equilibrium of the fund, with a confidence level of 95%. Of course  $a$  is independent on the absolute value of  $X$ , i.e. the liability value, and depends by given  $(\mu, \sigma)$  only on the exercise time of the liability  $T$ .

#### Analysing equation (4) brings first results:

- *a is always smaller than the real return  $\mu$ , even for an infinite time horizon  $T$  (see also Table 1);*
- *For a short-time horizon  $T$  and for confidence levels close to 100%, the discount rate (d.r.)  $a$  is significantly smaller than  $\mu$ .*

So far, however, the imposed confidence level is totally arbitrary. We will now show that it is quite conservative when it is chosen between 90% and 95%, or larger.

Table 1 shows for a confidence level of 90% ( $k\sigma=1.282\sigma$ ), that the d.r. is as expected significantly lower than the return.

Confidence level= 90%				
T (years)	15	25	50	100
(a) 2.30%/3.90%	0.93%	1.22%	1.52%	1.72%
(b) 5%/10%	1.19%	1.94%	2.69%	3.22%

Table 1. Discount rate when the imposed confidence level for assets being larger than the liabilities  $X$  is 90% for the two portfolios (a) 100% bonds (b) medium-risk 50% equities/50% bonds.

Another difficulty in using this approach can be that no information is provided on how much in assets is missing on average to match the liabilities (0.01%, 5%, 20%, etc.). This information is not obtained by the confidence level alone, because the probability distribution is not normal, and it has a long asymmetric tail. This information on missing assets is thus needed in addition.

To avoid these drawbacks, it is better to use a different approach, based on *option theory*. The idea is to use the Black-Scholes pricing formula for European options, to be exercised in  $t=T$ . Of course the pricing must be coherent with the present approach, and give identical results when the same confidence level is used.

#### 4. CALCULATION OF A PUT OPTION

A put option covers the risk that the price  $S(T)$ , i.e., the asset value at time  $T$ , falls below the strike price  $X$ , i.e., the liability value. The value of the option is equal to the expected value  $E[\ ]$  of the difference between those two prices, resp. values as follows:

$$\text{Put option price} = P = E[\max(0, X - S(T))] \quad (5)$$

The price of the put option  $P$  is given by the celebrated Black-Scholes formula, we write here for the put pricing, rather than for the call pricing, as usual:

$$P = S[1 - \Phi(d_1)] - Xe^{-rT}[1 - \Phi(d_2)]$$

$$d_1 = d_2 + \sigma\sqrt{T} \quad (6)$$

$$d_2 = \frac{1}{\sigma\sqrt{T}} \ln \frac{S}{Xe^{-rT}} - \frac{1}{2} \sigma\sqrt{T}$$

, where most variables have been defined before with the exception of:

-  $r$  is the risk-free rate of return. In our model,  $r = \mu$ , the expected return rate.

-  $\sigma$  represents the standard deviation of the assets, as in table 1.

-  $\Phi(d_1)$  represents the so-called ‘delta’, i.e. the derivative of the call-price with respect to the price of the assets.

-  $1 - \Phi(d_2)$  represents the probability of exercising the put option, i.e. the probability that  $S \leq X$ . In our interpretation, we have the probability that the assets will be inferior to the liabilities at time  $t=T$ . To make a link with the calculation in the previous section 3, this probability gives the confidence level for having sufficient assets for the funding.

The idea behind the calculation is as follows. Contrary to what was done in the previous section 3, a confidence level for having sufficient assets will not be used here. It is now imposed that, on average, the missing assets given by equation (5) cannot exceed a given percentage of the liabilities  $X$ .

Let us call ‘ $F$ ’ this percentage. Inserting equation (2) in the BS formula (6), one can solve the following implicit equation in the d.r. ‘ $a$ ’ by using the ‘Goal Seek’ function of EXCEL:

$$P(a)e^{\mu T} = FX \quad (7)$$

The put price is calculated at  $t=0$ , it is why to be compared with the missing percentage on the liability shown on the RHS of (7); it has to be forward discounted to  $t=T$  with the risk-free rate  $\mu$ .

Using the resulting ‘ $a$ ’ value, the current price  $S_0$  (= current asset value), can be calculated by equation (2).

As an example, use the 100%-bond portfolio, by imposing a funding ratio of 98% (2% in assets are missing) in  $T=15$  years, for an arbitrary  $X=100$ . It comes:

$$\text{d.r. } a = 1.52\%$$

$$S = 79.6$$

$$\Phi(d_1) = \text{delta} = 80.31\%$$

$$\Phi(d_2) = 75.86\% \rightarrow 1 - \Phi(d_2) = 24.14\%$$

$$\text{Call price} = C = 10.23$$

$$\text{Put price} = P = 1.42$$

It is clear that

$$F = Pe^{\mu T} = 2\%$$

So, although the missing assets represent on average only 2% of the liability, the confidence level is only 75.9%; the probability of exercising the put is 24.1%, which is a quite high probability.

## 5. A METHODOLOGY FOR DEFINING A DISCOUNT RATE (D.R.)

The valuation of the put option can thus serve as a basis for selecting a d.r., and for adjusting the funding level, as we now discuss. In the example handled in section 4, the following results are obtained:

- The imposed maximum error on the cost, i.e., the average cost excess with respect to the income is 2% on average. To take care of these 2% missing assets, the funding must be augmented by a premium of at least 2%;
- the d.r. on which this calculation is based comes out to be 1.52 %, though the expected return of the bond portfolio is  $\mu = 2.30\%$ .

These results are not yet satisfactory, however, for two reasons:

1. The choice of an average error of 2% on the stochastic cost is totally arbitrary, like the previous choice of a confidence level of 90% in section 3;
2. The 2% premium on funding only covers *on average* the missing assets.

With respect to topic 1, it is better to start with the choice of a value for the discount rate, e.g. 2%, and to evaluate the error to be used for the calculation of the premium. To be acceptable the d.r. choice must give a confidence level equal to at least 50% in the calculation of the put option. Otherwise the average assets available in T would come below the cost X, which is not acceptable.

We thus have a first set of necessary conditions:

$$a < \mu \quad (8a)$$

for having

$$\text{confidence level} > 50\% \quad (8b)$$

With respect to the second condition, we propose to proceed as now explained:

Fig. 1 shows the distribution of assets in  $t=T$ . The first condition imposes that the strike price X, i.e., the liability value, comes left to the average value of this distribution. The probability of exercising the put option in T is equal to (1- confidence level). It is equal to the surface below the distribution, to the left of X. This surface can be approximated by a rectangular triangle of equal surface, and identical center of gravity. Measuring the distance between X and the projection of the center of gravity on the asset axis, gives a value, which is equal to the put-option value in  $t=T$ , i.e.,  $Pe^{\mu T}$ , where P is the option price in  $t=0$ . This distance is equal to 1/3 of the length of horizontal edge of the rectangular triangle, as can be immediately

seen from the properties of the center of gravity in a triangle. It is thus a good idea to assimilate the left tail of the lognormal distribution to a triangle with identical surface. Three-times the option value is calculated as a premium for the funding. In this way, funding is also provided for the deficit corresponding to the leftmost edge of the equivalent rectangular triangle of figure 1. This gives the calculation of the funding premium:

$$\text{funding premium} = 3P \quad (8c)$$

The three conditions (8a) to (8c) complete the put-option methodology we have described for selecting a d.r. and applying a funding premium.

Table 2 shows the results for the two AA with (a) 100% bonds  $\mu = 2.30\% / \sigma = 3.90\%$ , and (b) a medium risk portfolio  $\mu = 5\% / \sigma = 10\%$ .

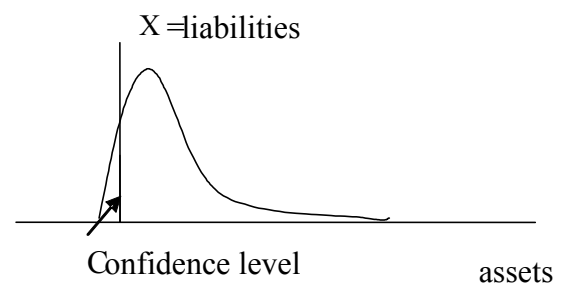


Figure 1. The log-normal asset distribution; the cost value (liabilities) X must be smaller than the average asset value. The confidence level is given by the surface below the distribution to the right of X. The surface to the left is equal to the probability of exercising the put option. The value of the latter can be estimated by the distance between X and the projected center of gravity of the described surface on the asset axis.

d.r.	2% $\mu = 2.30\%$ ; $\sigma = 3.90\%$				
T (years)	5	15	25	50	100
Premium (%)	8.4	12.4	14.3	16.4	17.0
Confidence level (%)	55.1	58.8	61.3	65.8	71.7

(a)

d.r.	2% $\mu = 5\%$ ; $\sigma = 10\%$				
T (years)	5	15	25	50	100
Premium (%)	10.8	8.7	6.3	2.6	0.5
Confidence level (%)	71.2	83.4	89.4	96.1	99.4

(b)

Table 2. Results of the determination of the discount rate, funding premium, and confidence levels for the two AA's (a) and (b).

In summary we observe from this table that:

- The results are according to (8a) to (8c). However though in both cases the confidence levels are increasing with T, this is a slow effect for the bond portfolio (a);
- The confidence levels for the bond portfolio (a) remains poor, even for larger T's with a value inferior to 75%. The confidence levels of the medium-risk portfolio (b) are by contrast satisfactory at the onset, and they are growing more rapidly to settle close to 100% for larger T values;
- The premium is increasing with T for portfolio (a); it is decreasing with T in the case of portfolio (b);
- It thus appears that the d.r.=2% is too large for a 100%-bond portfolio for all T's. By contrast this value might be accepted for the portfolio ( $\mu = 5\%$ ;  $\sigma = 10\%$ ) assuming in addition  $T \geq 15$  years.

This last point illustrates the fact that caution is needed before considering a suitable d.r.. Given an AA, acceptable results for each T may require a significantly smaller value than the expected return. Table 3 shows this to be the case for the 100%-bond portfolio: a = 1.8%, or smaller, would be preferred to a=2%.

d.r.	1.80%				
T (years)	5	15	25	50	100
premium	7.19%	9.33%	9.79%	9.24%	7.00%
confidence level	60%	66%	71%	78%	86%

Table 3. The d.r.  $a=1.8\%$  gives more satisfactory results than  $a=2\%$  with respect to premium and confidence level in the case of the 100%-bond portfolio ( $\mu = 2.3\%$ ;  $\sigma = 3.90\%$ )

In addition to the premium which has to be added to the funding this reduction of the d.r. induces an increase in the absolute value of the funding level (this level is equal to the NPV of the cost stream). The funding has thus to be increased by a factor  $\left[ \frac{(1+2\%)^T}{(1+a)} \right]$ , i.e., an increase of about 10 % for  $a=1.8\%$  and  $T=50$  years, a time horizon typical for radioactive waste disposal in geological host rocks.

## 6. PROVISIONAL CONCLUSIONS

In this paper we have presented an option methodology for calculating the discount rate to be used for funding environmental projects, assuming an extended range of time horizons. Note that in the example of radioactive-waste management this range can go from present to 100 years and more ahead.

Observing some of the results of the use of this approach we draw some first conclusions:

- The discount rate (d.r.) depends on the chosen AA adopted for managing the funding. It will grow much more slowly than the expected long-term returns attached to a particular AA, however;
- It results from that this first observation that it is often advisable to use a low d.r. value, even for riskier portfolios. This might underline that in actuarial calculations to approach a strategic AA (adapted to the liabilities and their distribution in time) a feedback process with the d.r. will develop, until coherent assumptions are obtained;
- Even with a low d.r. the risk of underfunding is not eliminated. We recommend including a premium, the value of which is equal to three-times the put-option price.

## REFERENCES

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